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# Quench-induced vortices in the symmetry-broken phase of liquid $^4\text{He}$

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**Abstract.** Motivated by the study of cosmological phase transitions, our understanding of the formation of topological defects during spontaneous symmetry breaking and the associated non-equilibrium field theory has recently changed. Experiments have been performed in superfluid  $^4\text{He}$  to test the new ideas involved. In particular, it has been observed that a vortex density is seen immediately after pressure quenches from just below the  $\lambda$  transition. We discuss possible interpretations of these vortices, conclude they are consistent with our ideas of vortex formation and propose a modification of the original experiments.

## 1. Introduction and overview

The formation of topological defects is generic to many physical systems, for example, superfluid  $^4\text{He}$  and  $^3\text{He}$ , superconductors and certain grand-unified theories which may be relevant to the early universe. The dynamics of vortex formation in all of these systems is intrinsically *non-equilibrium* due to the symmetry-breaking phase transition involved but despite their variety they can all be described by some form of field theory and their physical behaviour is very similar. Clearly there is some general feature of the behaviour of quantum fields out of equilibrium which is causing the universality of this behaviour. Such systems therefore provide a good arena for the study of non-equilibrium field theory, both experimentally and theoretically, since the defect density formed provides a good experimental test of the underlying field theory.

Here we are concerned with an experiment designed to test our ideas of what determines the defect density initially formed immediately after *second-order* symmetry-breaking phase transitions. The *mechanism*, sometimes called the Kibble mechanism, whereby topological defects, and vortices in particular, are formed [1] is understood and believed to be correct. It is not entirely certain, however, what sets the scale in this mechanism and thus determines the defect density immediately after the transition, although various scenarios have been proposed. It was originally thought that the defect density was determined when the system had cooled sufficiently far below the transition that thermal fluctuations were no longer important [2]. In other words it was thought that the string network was ‘frozen in’ at a temperature, the Ginzburg temperature, when there was no longer sufficient thermal energy

around for the field to fluctuate out of topologically stable configurations like strings. A criterion based on temperature cannot be wholly reliable because non-equilibrium physics is involved in the phase transition. An alternative, called the Zurek scenario [3], was suggested in which, due to the critical slowing down involved in second-order transitions, the field is unable to keep up with the quench during an adiabatic period of evolution around the transition. The length scale of the initial vortex network is then the correlation length of the field when it first comes back into equilibrium below the transition, which in this scenario is equal to the correlation length when the field went out of equilibrium above the transition.

An experiment, which we will refer to as the Lancaster experiment, designed to distinguish between these two alternative scenarios has been performed in superfluid  $^4\text{He}$  [4]. Preliminary results indicate the Zurek scenario to be correct, at least within its domain of applicability. There are, however, still some controversies and unresolved issues concerning whether or not the defect density might not perhaps have been formed by some means considered more conventional in condensed matter physics. In particular, there is an unexplained observation of vorticity after quenches from below the superfluid or  $\lambda$  transition. One of our aims is to explain this and thereby clarify the interpretation of the experiment. The experiment will then give us insight into the non-equilibrium behaviour of scalar fields during symmetry-breaking phase transitions.

## 2. The formation of vortices

Here we are interested solely in vortex production during second-order transitions which break a  $U(1)$  symmetry and produce vortices, as in superfluid  $^4\text{He}$  [5]. While there is no particularly good field theoretical model for superfluid  $^4\text{He}$ , to perform calculations we later assume a particular model [6] which exhibits all the generic features of a  $U(1)$  symmetry-breaking phase transition in which vortices are formed. We will also limit ourselves to the case of phase transitions sufficiently rapid that the Zurek scenario can be expected to be reasonably accurate.

The mechanism by which vortices are formed during such transitions is well understood [1]. In the symmetry-broken phase, above the phase transition, the order parameter is zero. As the transition is driven, by somehow removing energy from the system, the field begins to notice the central hill of the Mexican hat symmetry-breaking potential. Thus, the field tends to assume non-zero values and to increase in magnitude until it reaches its vacuum expectation value. There is no reason, however, for it to acquire the same phase everywhere and one therefore expects to have a patchwork of regions with differing phases. If on going round a loop in space which encloses several of these patches, the phase changes by a non-zero multiple of  $2\pi$ , then it follows that the field must vanish at least once within the loop and therefore that there must be at least one vortex passing through the loop. Thus defects are formed between regions of differing phase.

The question of what determines the initial vortex density, however, is less clear-cut. How many defects are formed must depend on the length scale over which the phase varies, or, roughly speaking, on the size of the domains of approximately constant phase. Amongst cosmologists, it was previously thought [2] that the relevant domain size was the thermal equilibrium coherence length of the field at the Ginzburg temperature,  $T_G$ . This is defined to be the temperature above which there is a significant probability for a thermal fluctuation of the field on a coherence length scale to unwind a defect by crossing the centre of the Mexican hat potential. In other words,

$$\xi^3 \Delta V / kT_G \approx 1$$

where  $\Delta V$  is the difference in free-energy density between the true and false vacua. The idea was that above this temperature any strings would be unwound by thermal fluctuations and only when the system cooled below this temperature could defects become quasi-stable or ‘frozen in’.

This argument is now believed to be wrong. The phase transition and the associated production of defects is an intrinsically non-equilibrium process, so it is incorrect to compute the defect density using the equilibrium field correlation length; indeed, it is strictly speaking not even possible to talk of a well defined temperature. Even were it possible always to define temperature, the Ginzburg temperature would not be relevant<sup>†</sup>. The physical meaning of  $T_G$  for the vortices is that this is the temperature above which they become rough: the thermal fluctuations in the vortex positions become larger than the vortex width. It is not correct to argue, however, as has often been done, that above this temperature strings can fluctuate in and out of existence. Above  $T_G$ , strings will start to wiggle randomly on small scales, and very small loops may appear and disappear, but a long string is unlikely to fluctuate into or out of existence as this would require a fluctuation on a scale of several coherence lengths. The small loops have very short lifetimes and would not normally survive long enough to be seen. In other words it is only coherent fluctuations on scales larger than the coherence scale of the field which are long-lived.

What is really needed is a truly non-equilibrium approach to the calculation of the defect density. In general, this is a difficult problem. However, in the case of a rapidly quenched second-order phase transition, the Zurek scenario, it is thought possible to make an estimate of the defect density produced comparatively simply [3]. The current understanding of this scenario is as follows.

In cosmological systems, there is a bound on the possible size of phase-correlated domains due to causality. A domain obviously cannot be larger than a horizon-sized volume and if one assumes on dimensional grounds that the defect density is roughly the reciprocal of the domain size squared, then this gives a lower bound on the defect density produced. In condensed matter systems, there is often an equivalent causal horizon which stems from the fact that correlations in the order parameter are not only causally bounded in principle by the speed of light, but in practice also by the speed at which interactions propagate in the system concerned. This is the basis of Zurek’s estimate of the vortex density in the Lancaster experiment in  $^4\text{He}$  and also of our work here.

Consider, for the sake of example, a condensed matter system like superfluid  $^4\text{He}$ , which starts in thermal equilibrium above the critical temperature and then undergoes a uniform temperature quench into the superfluid symmetry-broken phase. In real experiments a pressure quench is usually involved as it is difficult to produce uniform temperature quenches. For pedagogical clarity, however, we will consider first a temperature quench and only later show how to deal with a pressure quench. As the temperature is lowered and the phase transition proceeds, initially the relaxation rate of the field is greater than the rate at which the transition is being driven and the order parameter is able to keep up with the quench to maintain a roughly equilibrium configuration. It is therefore possible to talk about the temperature of the system during this early part of its evolution and also, since the system is almost in thermal equilibrium, to calculate the correlation length of the field as it increases early in the transition.

As the second-order transition is approached, however, for all modes of the order parameter field there will come a point during the transition when the relaxation of the

<sup>†</sup> The problem of trying to describe intrinsically non-equilibrium phenomena such as phase transitions, without using equilibrium concepts such as an effective potential, is very generic. For further comments see [7].

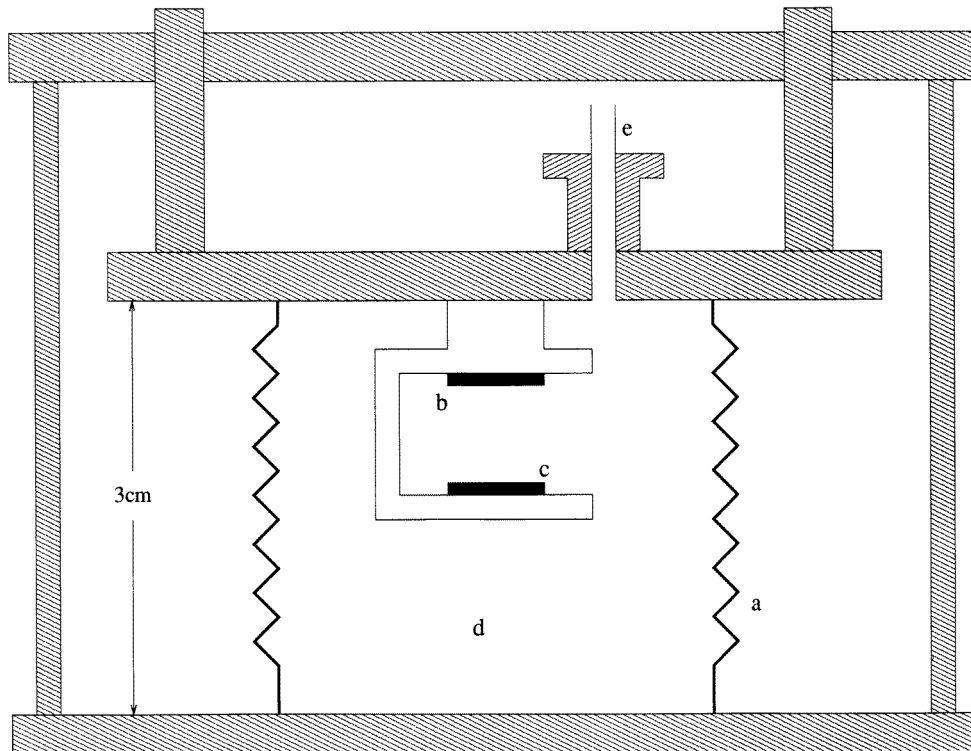
order parameter is not fast enough to keep up with the quench. This effect is particularly prominent during second-order transitions due to the critical slowing down of the low momentum modes of the field. For sufficiently rapid quenches, all modes of the scalar field will go out of equilibrium almost simultaneously and one can think of the field configuration at this instant being frozen in until at some stage below the transition the relaxation rate of the order parameter is again greater than the rate at which the transition is being driven. During this adiabatic phase of the transition, the dynamics will be roughly isentropic. The domain structure for the phase of the scalar field which gives rise to defects will then be that when the field configuration is frozen in so the defect density is calculated by finding the correlation length when the scalar field first goes out of equilibrium. In reality of course, modes do not go out of equilibrium instantaneously. Nevertheless, under the circumstances where the Zurek scenario is intended to be applied, this model seems to give a reasonably accurate estimate [4].

The Lancaster experiment in  $^4\text{He}$  discussed below uses a pressure rather than a temperature quench to induce the phase transition. This situation is experimentally simpler in that it avoids the difficulty of trying to induce a uniform temperature quench. We can assume that the process starts from an equilibrium state just above or just below  $T_c$ , and there is a rapid pressure quench from there to a state well away from the  $\lambda$ -transition at the lower pressure. In the spirit of the Zurek scenario it then seems reasonable to assume that the equilibrium configuration of the field at the original temperature and pressure is simply frozen in when the field goes out of equilibrium and hence determines the defect density measured.

### 3. The Lancaster experiment and its interpretation

There have been three experiments relevant to the study of non-equilibrium field theory in defect formation. The first involved a temperature-driven first-order phase transition in nematic liquid crystals [8] and was primarily intended to demonstrate the analogy between the behaviour of defect networks in nematics long after the phase transition when the symmetry has been broken almost everywhere and the scaling of cosmic string networks. Although it also had some relevance to the consideration of defect formation, it suffered from the experimental problem that it is difficult to produce a uniform temperature quench in a nematic over more than a few correlation volumes. The second experiment, performed in Lancaster [4], was specifically designed to test the Zurek scenario and exploit the analogy between a hypothetical grand-unified scale symmetry-breaking transition producing cosmic strings in the early universe and the  $\lambda$  transition in liquid  $^4\text{He}$ . Although this second experiment is inherently far more difficult, preliminary results seem to vindicate the Zurek scenario, subject to various caveats discussed below. The third and most recent experiment [9], involving superfluid  $^3\text{He}$ , also appears to vindicate the Zurek scenario.

The Lancaster experiment, which is shown schematically in figure 1, may be summarized as follows. A  $10^{-3}$  kg isotropically pure sample of liquid  $^4\text{He}$  is held at about 2 K in a bronze bellows a few centimetres in length. Both the pressure and temperature of the sample are recorded and the vortex density is measured indirectly by the attenuation of second sound between a heater–bolometer pair 4 mm apart. The bellows is compressed and the sample allowed to reach thermal equilibrium before being released and allowed to expand by 4 mm in a period of roughly 3 ms. Since the speed of first sound, which carries information about pressure and therefore the progress of the transition, is so much greater than that of second sound, which is related to the relaxation rate of the scalar field, it is possible to induce the phase transition simultaneously over many causally disconnected regions.



**Figure 1.** The Lancaster experiment (schematic—not to scale): a, phosphor-bronze bellows; b, bolometer; c, heater; d, isotropically pure sample of  $^4\text{He}$ ; e, sample filling tube.

After the pressure quench, the vortex density produced will decay over a period of a few seconds at a rate [10, 11, 18]

$$\frac{dL}{dt} = -\chi_2 \frac{\hbar}{m_{\text{He}}} L^2$$

where the Vinen parameter  $\chi_2$  is a dimensionless constant and it has been assumed that vortex creation due to the non-conservative interaction between the normal fluid and vortices already present is not significant. Although the whole apparatus vibrates for a few tens of milliseconds after the quench, thus initially swamping the electronic detection apparatus, and the decay of such a vortex density is rapid, using the defect attenuation of second sound, which travels at a few metres a second in the regime concerned, there is time to measure the decreasing vortex density before equilibrium is reached and the attenuation becomes constant.

There are several problems with this experiment, most obviously the effects of turbulence introduced by the walls of the bellows and the capillary used to fill the cell, the latter possibly being the more serious effect. Also, it is difficult to repeat exactly the same run many times as the precise isentrope followed during the quench can only be determined retrospectively. Although a pressure quench induced by allowing the bellows to expand produces a very uniform quench, the vibrations of the bellows mean that the electronics of the heater–bolometer system are swamped by noise for about 40 ms. It has also been commented that it is very difficult to prepare a sample of superfluid helium-4 without any

vorticity and that it is therefore not entirely clear what is causing the vorticity.

Regardless of the problems, however, the results are as follows. For quenches from above the lambda transition, the expected attenuation of the second sound signal is seen. Although at very early times after the transition, the electronics are swamped by spurious signals due to the vibration of the apparatus, it is straightforward to extrapolate although unfortunately only to a limit on the defect density formed of

$$n > 10^{13} \text{ m}^{-2}$$

where  $n$  is the vortex line density. The Zurek prediction is close to this limit. These results do allow us at least to determine that the Ginzburg temperature is not relevant. The Ginzburg temperature lies far below the final state to which the system is quenched, about half a degree Kelvin below the critical temperature, and cannot therefore be relevant to the formation of defects. The Zurek prediction is seemingly very good, but the presence of vorticity after quenches from just below the  $\lambda$ -transition casts doubt on the Zurek scenario as the only significant means of vortex production.

Similar quenches starting from thermal equilibrium significantly below the  $\lambda$ -transition, but still within the Ginzburg regime, do not show an equivalent attenuation and it is claimed that this rules out vortex production by motion between the fluid and the corrugated walls of the bellows [4]. If, however, the expansion which induces the pressure quench commences within about 10 mK of the transition, some comparatively short-lived attenuation *is seen* and indicates the presence of vortices although at a lower density than that produced by quenches through the  $\lambda$ -transition. It is this vorticity which concerns us here.

#### 4. Quenches from below $T_\lambda$

Clearly the defects which are observed after quenches starting from below the phase transition are not formed by the Kibble mechanism since the symmetry is already broken in the initial state. This forces us to question whether the vorticity created during quenches from above the phase transition is due to the Kibble mechanism or whether there is some additional means of forming defects which contributes to the vorticity created in transitions from both above and below the phase transition. The most obvious idea is to attribute the subcritical vortices to thermal fluctuations in the order parameter. In other words, so close to the phase transition the thermal defect density is sufficiently high that these defects live through the quench to be seen in the Lancaster experiment. The fact that such vortices would be almost all in the form of small coherence sized loops is irrelevant as they will all still attenuate second sound. Let us therefore estimate this thermal defect density. The first guess would therefore be that, although with a quench from a point already below the phase transition critical slowing down becomes less significant as the transition proceeds, rather than more important as was the case with the original Zurek scenario, for sufficiently fast quenches, the thermal vortex density is frozen in and ends up being the observed density. For more realistic quenches at slower rates, the system will be better able to keep up with the quench and will therefore end up closer to the perfect equilibrium situation with fewer vortices than would be predicted by this freezing-in argument. In other words, applying the Zurek scenario to quenches either from above or below the transition gives an upper limit on the vortex density.

In order to compute the density of vortices, we need some way to identify and count configurations of the field which have non-zero winding number and constitute vortices. If on going around a loop in space, the phase of the scalar field changes by some non-zero integral multiple of  $2\pi$ , then it is not possible for the field to be in the true vacuum state

everywhere within the loop. There must be at least one point within the loop where the field is in the false vacuum, that is where the field vanishes. This point can be thought of as the centre of the string and is the only gauge-invariant value of the field. Thus, in any gauge, the centre of a string must be a zero of the field. Although it is true that all strings have a zero at their centre, it is not necessarily true that all zeros of the field are associated with vortices. What is in fact required is the number density of zeros with large-scale winding around them.

Halperin has suggested a method of computing the topological line density  $\rho(\mathbf{r})$  or defect density [13, 14] defined by

$$\rho(\mathbf{r}) = \sum_n \int ds \frac{d\mathbf{R}_n}{ds} \delta^3[\mathbf{r} - \mathbf{R}_n(s)]$$

where  $ds$  is the incremental length along the line of zeros  $\mathbf{R}_n(s)$  ( $n = 1, 2, \dots$ ) and  $d\mathbf{R}_n/ds$  is a unit vector pointing in the direction which corresponds to positive winding number. Only winding numbers  $n = \pm 1$  are considered. Higher winding numbers are understood as describing multiple zeros. If  $dA$  is an incremental two-dimensional surface containing the point  $\mathbf{r}$ , whose normal is in the  $j$ th direction, then  $\rho_j(\mathbf{r})$  is the net density of strings or in other words the density of strings *minus* the density of antistrings on  $dA$ .

We shall only consider situations in which

$$\langle \rho_j(\mathbf{r}) \rangle = 0.$$

That is, we assume equal likelihood of a string or an antistring passing through an infinitesimal area. It follows that, in terms of the zeros of  $\Phi(\mathbf{r}) = \Phi_1(\mathbf{r}) + i\Phi_2(\mathbf{r})$ ,  $\rho_i(\mathbf{r})$  can be written as

$$\rho_i(\mathbf{r}) = \delta^2[\Phi(\mathbf{r})] \epsilon_{ijk} \partial_j \Phi_1(\mathbf{r}) \partial_k \Phi_2(\mathbf{r})$$

where  $\delta^2[\Phi(\mathbf{r})] = \delta[\Phi_1(\mathbf{r})] \delta[\Phi_2(\mathbf{r})]$ . The coefficient of the  $\delta$ -function in this expression is the Jacobian of the transformation from line zeros to field zeros. While the above expression is very good insofar as it counts only objects with winding which are genuinely defects, it is not always the easiest thing to calculate. In thermal equilibrium, however, the gradient terms take on an average value and Halperin's method is then entirely equivalent to counting the thermal expectation of the number of zeros as follows.

In our simplified method, which applies only to thermal equilibrium when the structure of the vortices is known, we count all the zeros of a smoothed field on a particular surface, regardless of sign. In thermal equilibrium at a temperature  $T$  below the phase transition, the expected number density of defects passing through a circular loop of radius  $R$  will be

$$\langle n \rangle = \frac{1}{\pi R^2} \langle \text{number of zeros of } \phi \rangle$$

where the angular brackets denote a quantum mechanical and thermal average which must ultimately be computed using some field theory describing  $^4\text{He}$ .

One advantage of considering the zeros of  $|\phi|^2$  is that this is a manifestly gauge-invariant quantity. To count the zeros, we look for regions where  $|\phi|^2$  is within some specified range of zero, namely  $|\phi| \leq \eta(T)/\sigma$ , where  $\eta(T)$  is the vacuum expectation value of the field in the broken phase—we may call this the region of hot phase. Ultimately we take the limit  $\sigma \rightarrow \infty$ . For small values of  $r$ , an individual vortex is described by

$$|\phi(r)| \approx \eta(T)r/\xi(T)$$

where  $1/\xi(T)$  is the temperature-dependent mass of the scalar particle which can later be calculated from the action [6]. Hence, assuming an isotropic distribution of the thermal



string and averaging over the possible orientations of such string, which give different areas of intersection with the disc of radius  $R$ , the area within which the hot-phase condition is satisfied around a single vortex is

$$A_{\text{string}} = \frac{2\pi\xi^2}{\sigma^2}$$

where the factor of two comes from the angular averaging process. It follows that

$$\langle n \rangle = \frac{1}{\pi R^2} \frac{\langle \text{area in hot phase} \rangle}{A_{\text{string}}}.$$

To find the area in the hot phase, we use a Gaussian window function  $\exp[-\sigma^2|\phi|^2/\eta^2(T)]$ . Thus the expected number density of zeros is

$$\begin{aligned} \langle n \rangle &= \frac{1}{\pi R^2} \lim_{\sigma \rightarrow \infty} \frac{\sigma^2}{\pi \xi^2(T)} \left\langle \int_{\text{loop}} d^2\mathbf{x} \exp\left(\frac{-\sigma^2|\phi|^2}{\eta^2(T)}\right) \right\rangle \\ &= \frac{1}{\pi \xi^2(T)} \lim_{\sigma \rightarrow \infty} \left\langle \sigma^2 \exp\left(\frac{-\sigma^2|\phi(\mathbf{0})|^2}{\eta^2(T)}\right) \right\rangle \end{aligned}$$

since  $\langle |\phi|^2 \rangle$  is translationally invariant.

Actually, we are not interested in all zeros. Small-scale fluctuations may lead to the appearance of several short-lived zeros within a particular vortex. To eliminate overcounting, we will smooth  $\phi$  on a scale comparable to the thermal coherence length. Indeed, the assumed behaviour of the field near a zero is correct only if the field is smoothed. Thus we avoid counting two zeros within a string width of each other as more than one string. We do this by replacing  $|\phi(\mathbf{0})|^2$  with

$$\int_0^\beta d\tau \int d^3\mathbf{x} \int_0^\beta d\tau' \int d^3\mathbf{y} \phi^*(\mathbf{x}) f(\mathbf{x}) \delta(\tau) \phi(\mathbf{y}) f(\mathbf{y}) \delta(\tau') \quad (1)$$

where

$$f(\mathbf{x}) = \left(\frac{1}{2\pi\zeta^2}\right)^{3/2} \exp\left(\frac{-|\mathbf{x}|^2}{2\zeta^2}\right).$$

Here  $\zeta$  is the length scale on which the field is smoothed. It is necessary to have a very good physical motivation for the choice of this scale. Later, it will be chosen to be of the order of the thermal correlation length  $\xi(T)$ . It need not *a priori* be equal to it, however. Since  $f(\mathbf{x})$  is strongly peaked within a thermal coherence length of the origin but still non-singular, this technique has the additional advantage of eliminating unphysical divergences in the thermal and quantum mechanical averages. On scales smaller than a coherence length, the quantum fluctuations will dominate over the thermal. In other words, on such scales the Heisenberg uncertainty principle is producing a large population of virtual defects which would cause a divergence in the number density without a cut-off. This is very similar to a population of virtual photons round an electron, although it is more obvious if one uses a dual theory of the transition in which the quanta are the vortices instead of the usual field theory where the quanta are related to helium atoms.

The number density of vortices can therefore be found by calculating the expected number of zeros of the scalar field on a hypothetical flat circular disc. This is close to the experimental procedure, where the defect density is measured by the attenuation of second sound, which is attenuated by any region of symmetry-unbroken phase. It should be noted that this method will not in itself give any information about the length distribution of the vortices.

Assuming that the smoothed scalar field does not vary very much over length scales comparable with the range of the pair potential  $V(\mathbf{x})$ , which experimentally is of the order of the typical atomic separation, we are able to use an action with the local form

$$S[\phi] = \int_0^\beta d\tau \int d^3\mathbf{x} \phi^*(\mathbf{x}) \left[ -\frac{1}{2m} \nabla^2 - \mu + \frac{\partial}{\partial \tau} + \lambda |\phi(\mathbf{x})|^2 \right] \phi(\mathbf{x}) \quad (2)$$

where  $\lambda = \tilde{V}(\mathbf{0})$  and we set  $\hbar = 1$ . SI units will be restored later in order to facilitate a comparison with experiment. Also, the term which looks like a chemical potential,  $\mu$ , is defined so that  $\mu = \mu_0 + V(0)/2$  where  $\mu_0$  is the genuine chemical potential of the helium atoms. This may be deduced from many-particle quantum mechanics with the sole additional approximation that the interactions may be represented locally. This may not be a particularly good approximation, however, since the helium atoms have a finite size and separation of the order of a few Ångströms, not much less than a typical coherence length of the field, thus placing a limit on how local the interaction can be. Other condensed matter systems, such as superfluid  $^3\text{He}$ , do not suffer from this problem. They do, however, exhibit a variety of defects which makes calculation significantly more complicated if no more difficult in principle.

### 5. Calculation of the thermal density

Consider first the calculation of the vortex density in thermal equilibrium in the symmetry-broken phase. Below the temperatures at which the phase transition actually occurs, we may expand the field  $\phi$  about a particular point on its vacuum manifold

$$\sqrt{2}\phi = \eta(T) + A(\mathbf{x}) + iB(\mathbf{x}).$$

Let us for the moment assume that we may use mean-field theory, and so rewrite the combinations of  $A$  and  $B$  thus:

$$\begin{aligned} A^3 &\approx 3\langle A^2 \rangle A \\ A^4 &\approx 6\langle A^2 \rangle A^2 - 3\langle A^2 \rangle^2 \\ A^2 B^2 &\approx \langle A^2 \rangle B^2 + A^2 \langle B^2 \rangle - \langle A^2 \rangle \langle B^2 \rangle \\ AB^2 &\approx A \langle B^2 \rangle \end{aligned}$$

where quantities such as  $\langle A^2 \rangle$  are later to be computed self-consistently. It is quite reasonable to object to this in that the region of interest is well within the Ginzburg regime in  $^4\text{He}$ , in other words less than half a degree Kelvin away from the transition. This region is defined to be precisely the region in which thermal fluctuations are important and mean-field theory is known to be a poor approximation in this case [15]. However, mean-field theory will give at least qualitatively the right behaviour and is unlikely to be inaccurate by *many* orders of magnitude. It turns out that this is sufficient for our purposes.

We may now use a path integral method to compute the number density of zeros. The integrand of the path integral contains the exponential factor from (1), with  $|\phi(\mathbf{0})|^2$  replaced by (1), as well as the exponential of the action (2). The integral factorizes, in the form

$$\int \mathcal{D}A \exp\{-S[A]\} \int \mathcal{D}B \exp\{-S[B]\} = I_A I_B \quad (3)$$

where

$$S[A] = \int_0^\beta d\tau \int d^3\mathbf{x} \int_0^\beta d\tau' \int d^3\mathbf{y} \left[ \frac{\sigma^2}{2\eta^2} (\eta^2 + A(\mathbf{x})A(\mathbf{y}) + 2\eta A(\mathbf{y})) f(\mathbf{x})f(\mathbf{y})\delta(\tau)\delta(\tau') \right. \\ \left. + \frac{A(\mathbf{x})}{2} \left( -\frac{1}{2m} \nabla^2 + \frac{\partial}{\partial\tau} + C_2 \right) \delta^3(\mathbf{x} - \mathbf{y})\delta(\tau - \tau') A(\mathbf{y}) \right] \\ - \int_0^\beta d\tau \int d^3\mathbf{x} C_1 A(\mathbf{x})$$

and

$$S[B] = \int_0^\beta d\tau \int d^3\mathbf{x} \int_0^\beta d\tau' \int d^3\mathbf{y} \left[ \frac{\sigma^2}{2\eta^2} (B(\mathbf{x})B(\mathbf{y})) f(\mathbf{x})f(\mathbf{y})\delta(\tau)\delta(\tau') \right. \\ \left. + \frac{B(\mathbf{x})}{2} \left( -\frac{1}{2m} \nabla^2 + \frac{\partial}{\partial\tau} + C_3 \right) \delta^3(\mathbf{x} - \mathbf{y})\delta(\tau - \tau') B(\mathbf{y}) \right]$$

where the constants,  $C_1$ ,  $C_2$  and  $C_3$  are defined as

$$C_1 = \lambda\eta[\eta^2 + 3\langle A^2 \rangle + \langle B^2 \rangle] - \mu\eta \\ C_2 = \frac{\lambda}{2}[3\eta^2 + 3\langle A^2 \rangle + \langle B^2 \rangle] - \frac{\mu}{2} \\ C_3 = \frac{\lambda}{2}[\eta^2 + \langle A^2 \rangle + 3\langle B^2 \rangle] - \frac{\mu}{2}.$$

These path integrals are of a standard well defined Gaussian type and since the non-perturbative physics is already in our equations, it does not matter that we expand about a particular point on the vacuum manifold. Before proceeding to do the path integrals, however, let us first consider the self-consistent calculation of the coefficients  $C_1$ ,  $C_2$  and  $C_3$ , or equivalently of  $\langle A^2 \rangle$  and  $\langle B^2 \rangle$ . To do this, we exploit the fact that the real scalar fields  $A$  and  $B$  must have a null expectation value. Viewing  $C_1 A$  as a term in the potential for the  $A$  field, it will be seen that the only way to achieve  $\langle A \rangle = 0$  self-consistently is to impose  $C_1 \equiv 0$ . This then enables us to calculate  $C_2 = \lambda\eta^2$ .

We also know that  $B$  is a Goldstone mode, which implies that  $C_3 = 0$ . An interesting corollary of the fact that  $C_1$  and  $C_3$  both vanish is that  $\langle A^2 \rangle = \langle B^2 \rangle$ . Physically, this means that when both modes have zero effective mass at the lambda point, the equipartition theorem forces them to have the same average energy.

The result of this determination is to greatly simplify the  $A$  and  $B$  integrals. The path integrals may now be evaluated directly as they are both Gaussian. We will normalize both by dividing by similar integrals with the parameter  $\sigma$  set to zero, thus obtaining the actual number density of vortices. Quantities in which  $\sigma = 0$  will be denoted by a zero subscript.

Consider first the  $B$  integral. This is of the form

$$I_B = \int \mathcal{D}B \exp(-\frac{1}{2} B K B) = (\det K)^{-1/2} \quad (4)$$

where we have adopted the integration convention that spatial and Euclidean time integrals are now implicit on adjacent fields and operators and

$$K(\mathbf{x}, \tau; \mathbf{y}, \tau') = \frac{\sigma^2}{\eta^2} f(\mathbf{x})f(\mathbf{y})\delta(\tau)\delta(\tau') + \left( -\frac{1}{2m} \nabla^2 + \frac{\partial}{\partial\tau} \right) \delta^3(\mathbf{x} - \mathbf{y})\delta(\tau - \tau').$$

The Fourier transform of this,

$$\tilde{K}(p, q) = \frac{\sigma^2}{\eta^2} \tilde{f}(\mathbf{p})\tilde{f}(\mathbf{q}) + \left( \frac{|\mathbf{p}|^2}{2m} + ip_0 \right) (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{q})\beta\delta_{p^0, q^0}$$

allows us to compute the ratio  $I_B/I_{B,0}$ . This gives

$$\begin{aligned} \left(\frac{I_B}{I_{B,0}}\right)^{-2} &= \det\left(\frac{K}{K_0}\right) \\ &= \exp\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \text{Tr}[K_0^{-1}(K - K_0)]^n \\ &= \exp\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{\sigma^2}{\eta^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{\beta} \sum_{r=-\infty}^{\infty} \frac{|\tilde{f}(\mathbf{p})|^2}{|\mathbf{p}|^2/2m + 2\pi ir/\beta}\right)^n \\ &= 1 + \frac{\sigma^2}{\beta\eta^2} \sum_{r=-\infty}^{\infty} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{|\tilde{f}(\mathbf{p})|^2}{|\mathbf{p}|^2/2m + 2\pi ir/\beta}. \end{aligned}$$

Using the known result

$$\frac{2x}{\pi} \sum_{n=1}^{\infty} \frac{1}{x^2 + n^2} = \coth(\pi x) - \frac{1}{\pi x}$$

we may rewrite the sum to give

$$\frac{I_B}{I_{B,0}} = \left(1 + \frac{\sigma^2 D_0}{\eta^2}\right)^{-1/2}$$

where

$$D_0 = \frac{1}{2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} |\tilde{f}(\mathbf{p})|^2 \coth\left(\frac{|\mathbf{p}|^2}{4mT}\right)$$

with

$$\tilde{f}(\mathbf{p}) = \exp\left(\frac{-\zeta^2|\mathbf{p}|^2}{2}\right).$$

If  $\zeta^2 \gg 1/mT$ , we obtain

$$\frac{I_B}{I_{B,0}} \approx \left(1 + \frac{\sigma^2 mT}{2\pi^{3/2}\eta^2\zeta}\right)^{-1/2}.$$

The corresponding integral for the  $A$  field is slightly more complicated and of the form

$$\begin{aligned} I_A &= \int \mathcal{D}A \exp\{-\frac{1}{2}AKA - bA - \frac{1}{2}\sigma^2\} \\ &= [\det K]^{-1/2} \exp(\frac{1}{2}bK^{-1}b - \frac{1}{2}\sigma^2) \end{aligned} \tag{5}$$

where again there is implicit integration on adjacent fields and operators and

$$K(\mathbf{x}, \tau; \mathbf{y}, \tau') = \frac{\sigma^2}{\eta^2} f(\mathbf{x})f(\mathbf{y})\delta(\tau)\delta(\tau') + \left(-\frac{1}{2m}\nabla^2 + \frac{\partial}{\partial\tau} + \lambda\eta^2\right)\delta^3(\mathbf{x} - \mathbf{y})\delta(\tau - \tau')$$

$$b(\mathbf{x}, \tau) = \frac{\sigma^2 f(\mathbf{x})\delta(\tau)}{\eta}.$$

The required result is

$$\frac{I_A}{I_{A,0}} = \left[\det\left(\frac{K}{K_0}\right)\right]^{-1/2} \exp(\frac{1}{2}bK^{-1}b - \frac{1}{2}\sigma^2).$$

The determinantal factor follows as for the  $B$  integral:

$$\det\left(\frac{K}{K_0}\right) = 1 + \frac{\sigma^2}{2\eta^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} |\tilde{f}(\mathbf{p})|^2 \coth\left[\frac{|\mathbf{p}|^2}{4mT} + \frac{\lambda\eta^2}{2T}\right].$$

The exponential factor involves more extensive manipulations:

$$\begin{aligned} bK^{-1}b &= \frac{\sigma^2 f}{\eta} \left( K_0^{-1} - \frac{\sigma^2 f K_0^{-1} f}{\eta^2 + \sigma^2 f K_0^{-1} f} K_0^{-1} \right) \frac{\sigma^2 f}{\eta} \\ &= \frac{\sigma^4 D}{\eta^2 + \sigma^2 D} \end{aligned}$$

where

$$D = f K_0^{-1} f = \frac{1}{2} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} |\tilde{f}(\mathbf{p})|^2 \coth \left[ \frac{|\mathbf{p}|^2}{4mT} + \frac{\lambda\eta^2}{2T} \right].$$

Substituting this into the expressions for the normalized  $A$  integral above yields

$$\frac{I_A}{I_{A,0}} = \left( 1 + \frac{\sigma^2 D}{\eta^2} \right)^{-1/2} \exp \left[ \frac{-\eta^2 \sigma^2}{2(\eta^2 + \sigma^2 D)} \right].$$

Combining both of the factors (5) and (4) into (3) gives the final expression for the number density of vortices passing through a unit area at temperature  $T$  as

$$\langle n \rangle = \lim_{\sigma \rightarrow \infty} \frac{\sigma^2}{2\pi \xi^2(T)} \left( 1 + \frac{\sigma^2 D_0}{\eta^2} \right)^{-1/2} \left( 1 + \frac{\sigma^2 D}{\eta^2} \right)^{-1/2} \exp \left[ \frac{-\eta^2 \sigma^2}{2(\eta^2 + \sigma^2 D)} \right].$$

On taking the limit, we obtain the ultimate result

$$\langle n \rangle = \frac{\eta^2(T)}{2\pi \xi^2(T)(D_0 D)^{1/2}} \exp \left( \frac{-\eta^2(T)}{D} \right). \quad (6)$$

The integrals in  $D$  and  $D_0$  do not have any closed analytic form but are trivial numerically. However, if the temperature is large in the sense that  $T \gg 1/2m\xi^2$  or in other words,  $T \gg \mu = \lambda\eta^2$ , which is true well within the region of interest, the Ginzburg regime, then (6) reduces to the approximate form

$$\langle n \rangle = \frac{\eta^2(T)\zeta}{\pi \xi^2(T)mT} \exp \left( \frac{-\eta^2(T)\zeta}{mT} \right). \quad (7)$$

This result depends strongly on the smoothing scale  $\zeta$ . Conventionally in quantum field theory, one expects first to regulate an apparently divergent result by introducing a cut-off scale and then to show that the physically observable quantity is not dependent on this scale by renormalizing. In the current context, however, the physically relevant quantity should depend on a cut-off for two reasons. Firstly there is an intrinsic cut-off in the problem due to the discrete nature of the superfluid. Clearly, it is not sensible to talk about superfluid flow forming a vortex on scales smaller than this. In addition, however, we are only interested in coherent flows of the superfluid which are sufficiently long-lived to be seen by, for example, scattering experiments like that involving second sound described above. In other words we are only interested in flows of the superfluid on scales larger than the coherence length. There will indeed be fluctuations on scales smaller than this, which have winding number and zeros, but they are not relevant to the Lancaster experiment as this sees vortices only on longer time-scales. This is easiest to understand in terms of a classical analogy with a hurricane. Like a quantum vortex in superfluid  $^4\text{He}$ , a hurricane is a large-scale coherent motion of a fluid. One can clearly not talk about vortex or hurricane-like motions on scales less than the size of the particles making up the air. There will, however, be small-scale eddies on scales less than the large-scale coherent rotation of the whole hurricane. When counting hurricanes, however, it would not be considered even vaguely sensible to include the small-scale, short-lived eddies within the main hurricane. For this reason, we choose

our smoothing scale  $\zeta$  to be the thermal coherence length in our theory,  $\xi(T)$ . On restoring the factors of the Boltzmann and Planck constants, this leaves the following expression for the thermal vortex density:

$$\langle n \rangle = \frac{1}{\pi \xi^2} \frac{\eta^2(T) \xi(T)}{k_B m T} \exp\left(\frac{-\eta^2(T) \xi(T)}{k_B m T}\right). \quad (8)$$

This is not surprising: the energy of a correlation-sized loop of string is of order  $\eta^2 \xi / m$ , so the exponent here is essentially the expected Boltzmann factor.

However, the above calculation of the defect density depended crucially on the use of mean-field theory to decouple the Goldstone and Higgs modes of the scalar field and to force the integrals into Gaussian form. Although this is perfectly valid well below the transition, outside the Ginzburg regime, as we have already noted, it is not applicable to the main region of interest near the  $\lambda$  point. This is the major problem with our method of calculation. It will turn out, however, that even with the most optimistic assumptions concerning the superfluid density and coherence length, that the thermal defect density is too small by at least a few and possibly by many orders of magnitude to be the sole explanation for the vorticity observed after subcritical quenches. Mean-field theory is therefore sufficient for our purpose.

As a somewhat academic point we could, in principle, try to use the renormalization group to extrapolate into the region of interest. In the current context, however, it is not entirely clear how to go about applying these techniques. Physically, one expects that the form of the result (7) is unlikely to change discontinuously on reaching the Ginzburg regime and is likely always to be of the form

$$\langle n \rangle = \frac{\eta^2(T)}{\pi \xi^2(T)} \exp\left(\frac{-\eta^2(T)}{\langle A^2 \rangle_f}\right) \langle A^2 \rangle_f^{-1/2} \langle B^2 \rangle_f^{-1/2}$$

but with different expressions for  $\langle A^2 \rangle_f = D$  and  $\langle B^2 \rangle_f = D_0$ . One might therefore try to merely replace the coherence length and order parameter with renormalization group improved values.

In this context, a very interesting point emerges from (7) if we set  $\zeta = \xi(T)$  and consider the limit as  $T \rightarrow T_c$ . In mean-field theory,  $\eta^2 \xi \propto \epsilon^{1/2}$ , where  $\epsilon$  is the reduced temperature  $\epsilon = 1 - T/T_c$ , so the number of vortices per correlation area would tend rather rapidly to zero as  $T \rightarrow T_c$ . On the other hand, in the renormalization group,  $\eta^2 \xi \propto \epsilon^{2\beta - \nu}$ . But  $2\beta$  is actually very nearly equal to  $\nu$  (they would be exactly equal if the correlation-function critical exponent  $\zeta$  were zero, or equivalently if  $\delta = 5$ , in the usual notation). The best estimate is  $2\beta - \nu \approx 0.03$ . This means that until we get extremely close to the critical point, the number of vortices per correlation volume would remain of order one.

It is, however, difficult to decide what is the most honest, consistent thing to do. Let us make the obvious assumption that the vortex density seen immediately after subcritical quenches is just the frozen-in thermal density. For quenches from within a few mK of the transition, starting at a relative temperature of  $\epsilon = 1 - T/T_c = 10^{-3}$ , the thermal vortex line density calculated in the above mean-field scheme is of order  $\langle n \rangle \approx 10^5 \text{ m}^{-2}$ . Even closer to the transition at  $\epsilon = 10^{-4}$  the density is still only about  $10^{10} \text{ m}^{-2}$ . While this is consistent in the sense that mean-field theory has been used throughout, there is some justification for trying to use renormalization group improved values.

The other obvious thing to try is to simply substitute experimental values for the required quantities [16–18], using the molar volume as a function of pressure and the superfluid fraction to extract the vacuum expectation value of the order parameter and tabulated values

for the coherence length. Even with optimistic values for these parameters, the thermal defect density in the region of interest is still only of the order of  $10^{10} \text{ m}^{-2}$  at best.

One's first thought is that perhaps the mean-field approximation is so bad in the regime of interest, well above the Ginzburg temperature, as to severely underestimate the thermal defect density. Although the mean-field approximation is certainly not good in the regime of interest, it is hard to believe that it is in error by orders of magnitude, so our estimation of the thermal density seems to rule out the thermal vortex population as the sole source of vorticity from sub-lambda quenches. There are two possibilities, although these are not mutually exclusive. Either the thermal density acts as a seed and is magnified in some way or else there is some other source of vorticity.

## 6. Alternative explanations of the sub-critical vortex density

Let us first consider other sources of vorticity. The most likely source is the flows which are generated during the pressure quench by either the walls of the bellows or the capillary used to fill the bellows in the first place. The phosphor-bronze bellows are corrugated in order to allow for the compression and subsequent release of the sample chamber which produces the pressure quench. The capillary used to fill the sample chamber has to be long to ensure thermal isolation and has a valve on the end furthest from the opening into the sample. Thus during the pressure quench the column of fluid inside the capillary expands and about 20% of its volume is forcibly squirted into the chamber [19].

An idea due to Zurek is as follows [20]. The difference in the line density of vortices produced in quenches from very close to the transition and the density formed from slightly further away from the transition might be due to the different energy cost of creating vortices. It takes less energy to create a unit length of string near the transition since the vortex energy per unit length decreases with the superfluid density. The nearer the  $\lambda$ -transition the system is, the lower the superfluid density and hence the lower the tension of a vortex and the easier it is to form a unit length of one. The line density of vortices created in this way would then be proportional to the reciprocal of the superfluid density, or including logarithmic terms [20]

$$\langle \rho_s n \rangle_{\text{stir}} \propto (-\ln(\langle n \rangle_{\text{stir}} \xi^2))^{-1}.$$

This ought to be testable by comparing subcritical quenches with differing starting points. Since this argument does not depend in any way on the microphysics of superfluids, it is very robust but consequently it also fails to provide any insight into the details of what is actually going on and makes it difficult to ascertain whether the Zurek scenario is still valid in this regime.

Given the presence of such flows during the pressure quench, however, one might speculate that they interact with the thermal density of vortices which is already present. The phenomenology of this is well understood. Also, the interaction of a superflow with thermally generated vortices is well known in the context of the thermal generation of turbulence in a superfluid and is known by the acronym 'ILF' after Iordanskii, Langer and Fisher [21]. The presence of a filling capillary in the middle of the bellows is guaranteed to generate a superflow during the rapid pressure decrease of the quench and this capillary is probably far worse than the walls in terms of flow generation. Although the walls, in principle, allow vorticity to be generated by their motion relative to the superfluid, the rapid decompression of the superfluid in the capillary during the quench causes a superflow from the capillary.

Let us assume that our estimate of the thermal density of vortices just below the transition is reasonably accurate, at least to within a factor of ten or so. Since they are a thermal equilibrium population, these vortices will almost all be in the form of small coherence length sized loops and will move through the superfluid with a velocity given by [21]

$$v_i = \frac{\kappa}{4\pi R} \left[ \ln \left( \frac{8R}{a} \right) - \frac{1}{2} \right]$$

where  $\kappa = h/m \approx 10^{-7} \text{ m}^2 \text{ s}^{-1}$ ,  $R$  is the radius of a loop of string and  $a$  is the vortex core parameter first introduced by Feynman. When the decompression occurs, there will be superflows within the bellows. While the walls may perhaps not be terribly effective in generating these, it is guaranteed that superfluid will get forced out of the end of the filling capillary producing a superflow directly through the middle of the thermal vortex population. One can estimate  $v_s$ , the velocity of the superflow out of the capillary, from the change in pressure,  $\Delta P$  and the density of the liquid helium inside the capillary.

$$V \Delta P = \frac{1}{2} (\rho V) v_s^2.$$

For the change in pressure of about  $10^6 \text{ Pa}$  actually used this gives a velocity of roughly the critical superflow or slightly below, so it is possible that vorticity is generated by supercritical flows. However, since vorticity is not generated unless the quenches are sufficiently close to the transition, it is safe to assume that supercritical flows alone are not responsible for vortex formation and that the only possible explanation for the vorticity formed during quenches through the transition is that already described.

Even if the flow is not supercritical, however, the effect of a superflow on a population of vortex loops is to expand loops greater than a certain size with an appropriate velocity relative to the superflow according to

$$\frac{dR}{dt} = \left[ \frac{\gamma}{\rho_s \kappa} \right] (v_s - v_i).$$

Thus loops with sufficiently large radii that their velocity is less than the velocity of the superflow and also oriented so that some component of their velocity is parallel to the superflow, or roughly half the loops if the distribution is isotropic, will grow in the flow. This could result in the magnification of the thermal vortex line density.

For a thermal population of vortex loops, however, one expects almost all of the loops to be of a size roughly equal to the thermal coherence length. Sufficiently far from the transition, the vortex core parameter  $a$ , equal to the thermal coherence length, is only of the order of  $10^{-8} \text{ m}$  and there will be almost no loops sufficiently large, and therefore sufficiently slow in comparison with the superflow, to be expanded. Near the transition, however, the coherence length grows increasingly large and thus the velocity of the typical coherence sized loop becomes smaller and smaller as one approaches the phase transition. Sufficiently close to the transition, even coherence sized loops will have velocities slower than the superflow and will be expanded.

Assuming that the number density of loops of a given length decays fairly steeply with length, like  $l^{-5/2}$  or exponentially for example, it is a reasonable approximation to say that almost all of the string is in coherence length sized loops. Also, the density of thermal vortices is only significant near the transition. Thus, there will be almost no amplification of the thermal density by the superflows in the fluid unless the initial subcritical temperature is such that coherence sized loops move slower than the superflow velocity or in other words

$$\frac{\kappa}{4\pi \xi(T)} (\ln(8) - 0.5) < \sqrt{\frac{\Delta P}{\rho}}$$



or alternatively

$$\frac{T}{T_c} > 1 - \left( \frac{4\pi v_s \xi_0}{\kappa [\ln 8 - 0.5]} \right)^2.$$

In this scenario one would expect a fairly sharp temperature above which quenches start to produce significant vorticity as opposed to the simple stirring scenario in which one expects a gradual logarithmic increase in the vorticity. Also, the fractional increase in the vortex density or fractional amplification in this case is roughly

$$\frac{\Delta n}{n} \approx \frac{1}{R} \frac{dR}{dt} \Delta t \approx \frac{B(v_s - v_i)}{R} \Delta t \approx \frac{\Delta t v_s \epsilon^{1/2}}{\xi_0} \left[ 1 - \frac{\kappa \epsilon^{1/2}}{10 \xi_0 v_s} \right]$$

where  $\epsilon$  is the relative temperature  $1 - T/T_c$  and  $B \approx \gamma/\rho\kappa$  is a dimensionless parameter of order 6 in the region of interest. Between  $\epsilon \approx 10^{-3}$  and  $\epsilon \approx 10^{-4}$ , this amplification factor changes by only a factor of  $\sqrt{10}$ , being of order  $10^6$  or  $10^7$ . Using both experimental and renormalization group improved values, the thermal seed density varies only slowly in the regime of interest, although the density does vary by a factor of roughly  $10^5$  if mean-field critical exponents are used. It might therefore be expected that one would see a well defined temperature at which the amplification switches on.

In addition, if ILF is the dominant process, since conformal invariance demands that there are no vortices actually at the transition, as is consistent with the mean-field calculation above (7), there ought also to be an initial temperature above which no vorticity is observed following the pressure quench. However, this temperature may turn out to be so close to the critical temperature as to be unobservable.

## 7. Summary and conclusions

We have used an equilibrium finite temperature field theoretical description of superfluid  $^4\text{He}$  to predict the thermal density of vortex loops in the superfluid phase. This has been used in an adiabatic approximation to predict the vortex density produced in the Lancaster experiment for quenches from below the  $\lambda$ -transition. Although mean-field theory is a very poor approximation in the region of interest, it is unlikely to be inaccurate by more than a few orders of magnitude at most. Thus the fact that a mean-field theory estimate of the thermal vortex density below the transition gives a value several orders of magnitude below that observed implies that the vorticity observed is not simply the thermal population. Either the thermal population is amplified in some way or there is some other significant form of vorticity production. Here we have argued that the thermal density could be amplified by a superflow produced from the capillary which is used to fill the experimental cell. This is entirely analogous to the intrinsic thermal nucleation of defects known as the ILF process. The experimental signature of this scenario would be a sharp cut-off temperature such that quenches starting below this produce too little vorticity to be observed. There should also be an upper cut-off although this might be impossible to observe.

There are, however, problems. Firstly although it is difficult to come up with an alternative scenario with such copious vortex production, there are experimental problems which make it difficult to entirely eliminate other methods of vortex production. Secondly there is the theoretical problem that even if mean-field theory were a good approximation, the field theoretical description of  $^4\text{He}$  is inaccurate since the coherence length is never very much greater than the interatomic spacing.

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